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Ubiquitous log odds: a common representation of probability and frequency distortion in perception, action, and cognition

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In decision from experience, the source of probability information affects how probability is distorted in the decision task. Understanding how and why probability is distorted is a key issue in understanding the peculiar character of experience-based decision. We consider how probability information is used not just in decision-making but also in a wide variety of cognitive, perceptual, and motor tasks. Very similar patterns of distortion of probability/frequency information have been found in visual frequency estimation, frequency estimation based on memory, signal detection theory, and in the use of probability information in decision-making under risk and uncertainty. We show that distortion of probability in all cases is well captured as linear transformations of the log odds of frequency and/or probability, a model with a slope parameter, and an intercept parameter. We then consider how *task* and *experience* influence these two parameters and the resulting distortion of probability. We review how the probability distortions change in systematic ways with task and report three experiments on frequency distortion where the distortions change systematically in the same task. We found that the slope of frequency distortions decreases with the sample size, which is echoed by findings in decision from experience. We review previous models of the representation of uncertainty and find that none can account for the empirical findings.

Keywords: log odds, subjective probability, probability distortion, frequency estimation, decision-making, uncertainty

E . I **Figure 1A** (A , 1953). A E

Figure 1A . A E , , , , .

1

(G , 1999; L , 2000), (, 1966; B M , 1977; ., 1990), (A , 1953; L ., 1978), (., 2009, 2011).

Figure 1B (K , 1992). D (G , 1999; L , 2000) (B M , 1977; ., 2009)

(K , 1992).

. G (1999) fi .

/ . fi ,

$$\mathrm{L}\left(\pi\left(p\right)\right) = \gamma\,\mathrm{L}\left(p\right) + (1-\gamma)\,\mathrm{L}\left(p_0\right) \tag{1}$$

p / , $\pi(p)$ / ,

$$\mathrm{L}\left(p\right) = \frac{p}{1-p} \tag{2}$$

(B , 1949) (B , 1944). - (**Figure 2**).

γ E . 1 p_0 fi - , p . , $p = p_0$,

$$\mathrm{L}\left(\pi\left(p_0\right)\right) = \gamma\,\mathrm{L}\left(p_0\right) + (1-\gamma)\,\mathrm{L}\left(p_0\right) = \mathrm{L}\left(p_0\right). \tag{3}$$

1 . - - . ,A (1953) - - .

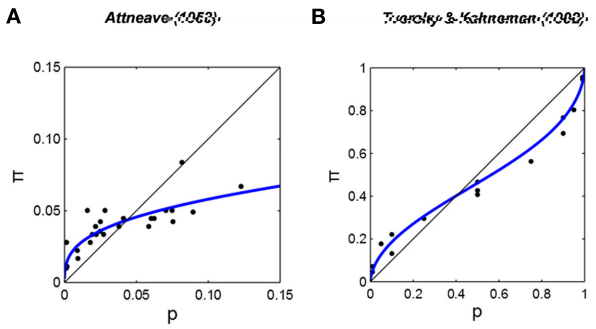

FIGURE 1 | S-shaped distortions of frequency estimates. (A) Estimated relative frequencies of occurrence of English letters in text plotted versus actual relative frequency from Attneave (1953). **(B)** Subjective probability of winning a gamble (decision weight) plotted versus objective probability from Tversky and Kahneman (1992). R^2 denotes the proportion of variance accounted by the fit.

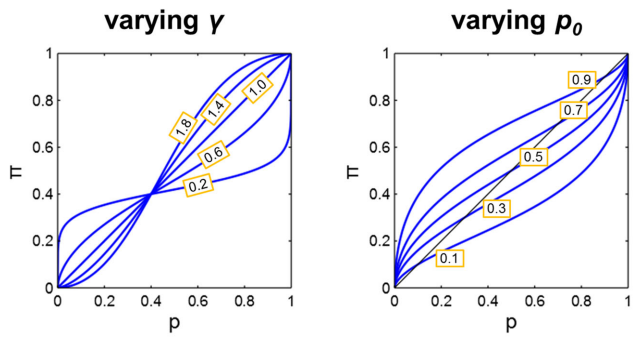

FIGURE 2 | Demonstration of the effects of varying the parameters γ and p_0. The parameter p_0 in the LLO function is the "fixed point" of the transformation, the value of p which is mapped to itself. The parameter γ, is the slope of the linear transformation on log odds scales, and on linear scales, is the slope of the curve at the crossover point p_0. Left: p_0 fixed at 0.4 and γ varied between 0.2 and 1.8. Note that the line at $\gamma = 1$ overlaps with the diagonal line, i.e., no distortion of probability. Right: γ fixed at 0.6 and p_0 varied between 0.1 and 0.9.

L () , $\pi(p_0) = p_0$. p_0 *crossover point.*

I **Figure 2** - , π p . 0 0, 1 1, p_0 p_0. A (p_0, p_0), γ. $\gamma = 1$, $\pi(p) = p$, , , . $\gamma > 1$ $0 < p_0 < 1$ - . $0 < \gamma < 1$ $0 < p_0 < 1$ - - . p_0 0 1, - . , - , (, 1966). I , (G E , 1987; - F , 1995; G , 1999). A -

fi K (1979) A (A , 1953). F G (1999) LL . LL J - . (1998) , , , LL (L , 2000).

. I L H J D - , fi LL - / . p π fi - . (γ p_0) - -fi (R^2) LL fi . γ p_0 : LL ? γ p_0? .

fl γ p_0. C C ? . - LL fi fl γ p_0. fi , (H ., 2004; H ., 2010). A - / , fi .I A D , fi L H J D C C ? I LL H , - / . LL - fl .

UBIQUITOUS LOG ODDS IN HUMAN JUDGMENT AND DECISION

J / fi LL γ p_0. I fi , - J / / . LL γ p_0. B . LL fi . , γ p_0 .

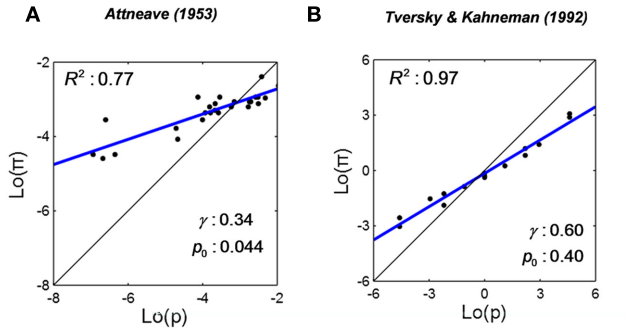

FIGURE 3 | Linear in log odds fits: frequency estimates. The two data sets in **Figures 1A,B** are re-plotted on log odds scales as **(A,B)**, respectively. The blue line is the best-fitting LLO fit. R^2 denotes the proportion of variance accounted by the fit. The S-shaped distortions of frequency/probability on linear scales in **Figures 1A,B** are well captured by the LLO fits.

FREQUENCY ESTIMATION

A (1953) - . I , E (**Figure 1A**). fi 63% , LL fi **Figure 3A** 77% .

( ) 0.15. I , $\hat{p}_0$, 0.044, A (1953) 1/26 (=0.039), . . A L . (1978). - 41 , fl , , (M A). - . - . E (1000) , M A (50000). ( ) , (2.05×10^8) , p π. fi - (. ., fl), LL . **Figure 4A** LL fi . I , - (. .,). LL - , . (1990), - LL . E . - . (1990) . **Figure 4B** LL fi .

CONFIDENCE RATING

C fi . F , G . (1991), - fi ? ( ) B ( ) A fi : 50, 51 60, 61 70, 71 80, 81 90, 91 99, 100% fi . 51 60% 55% fi , . C , fi , π. , p, fi fi fi fi . - G . (1991) **Figure 6** **Figure 5A**. γ LL fi . , ( ) - ( ). A . . G . (1991) fi . LL fi- . M G . (2004) - fi . - **Figure 5B**.

DECISION UNDER RISK OR UNCERTAINTY

A . K - (1979) J - , ... [2], - . B , (π) fi (p) . I **Figures 1B 3B**, - K (1992) . LL fi 97% , $\gamma = 0.60$ $p_0 = 0.40$. - - . A , G (1999) .

[2] . I , .

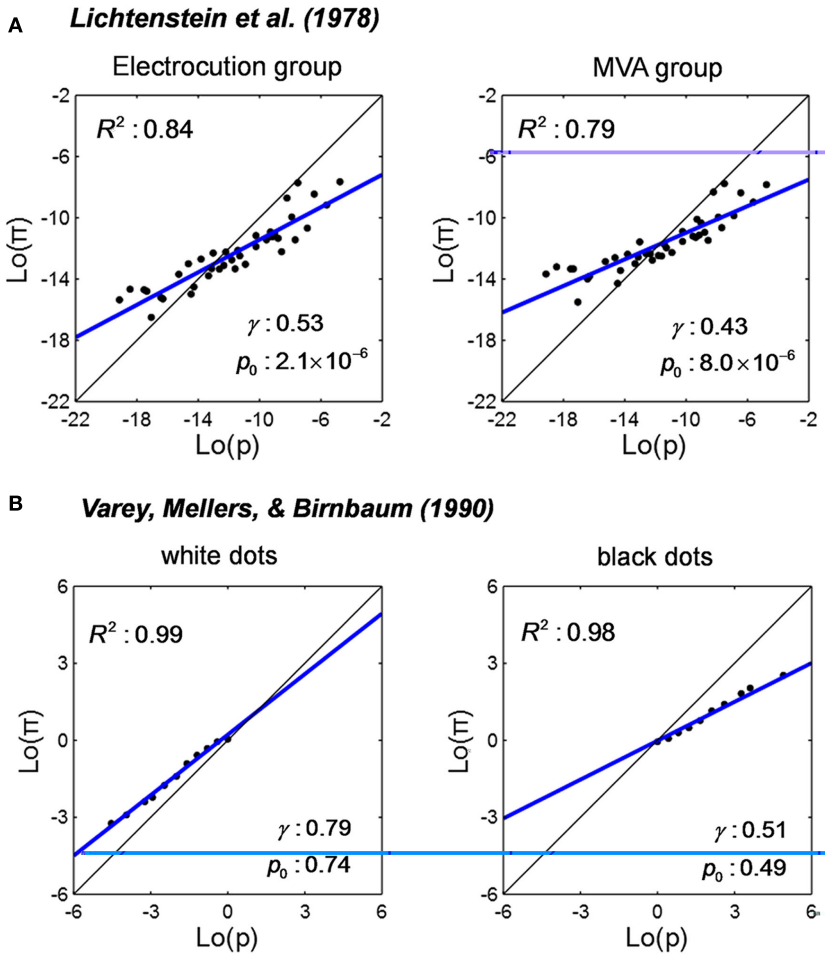

FIGURE 4 | Linear in log odds fits: frequency estimates from memory or perception. Estimated relative frequency is plotted against true relative frequency on log odds scales and fitted by the LLO function. Black dots denote data. The blue line denotes the LLO fit. R^2 denotes the proportion of variance accounted by the fit. **(A)** Estimated frequency of lethal events from Lichtenstein et al. (1978). Participants were asked to estimate the number of occurrences of different causes of death per year in the US. The actual frequency of one cause was provided as a reference for participants to estimate the frequencies of the other causes. The relative estimated and actual frequencies in the plot were the frequencies divided by the then US population. Left: when the frequency of Electrocution (1000) was given as reference. Right: when the frequency of MVA (motor vehicle accident, 50000) was given as reference. **(B)** Estimated frequency of visual stimuli from Varey et al. (1990). The task was to estimate the relative frequency of black or white dots among a visual array of black and white dots. The proportion of black dots was larger than the proportion of white dots. Two groups of participants respectively estimated the relative frequency of white dots (small p) and black dots (large p). Left: the white dots group ($p \leq 0.5$) was estimated. Right: the black dots group ($p \geq 0.5$) was estimated.

Figure 6A. E

γ 0.17 0.82, 0.30.

p_0 0.26 0.98,

0.46.

F (1995)

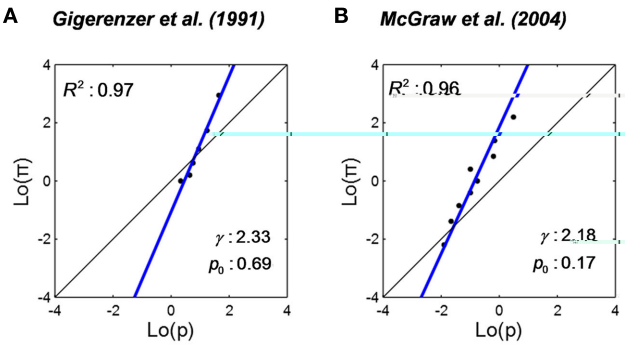

FIGURE 5 | Linear in log odds fits: confidence rating for cognitive and motor responses. Estimated probability of being correct or successful is plotted versus the actual probability on log odds scales and fitted by the LLO function. Black dots denote data. The blue line denotes the LLO fit. R^2 denotes the proportion of variance accounted by the fit. **(A)** Estimated probability of being correct in general-knowledge questions from Gigerenzer et al. (1991). Participants first chose an answer for two alternative general-knowledge questions and then indicated the probability that the answer was correct. **(B)** Estimated probability of success in basketball shooting from McGraw et al. (2004). Participants rated their probability of success before each basketball shot.

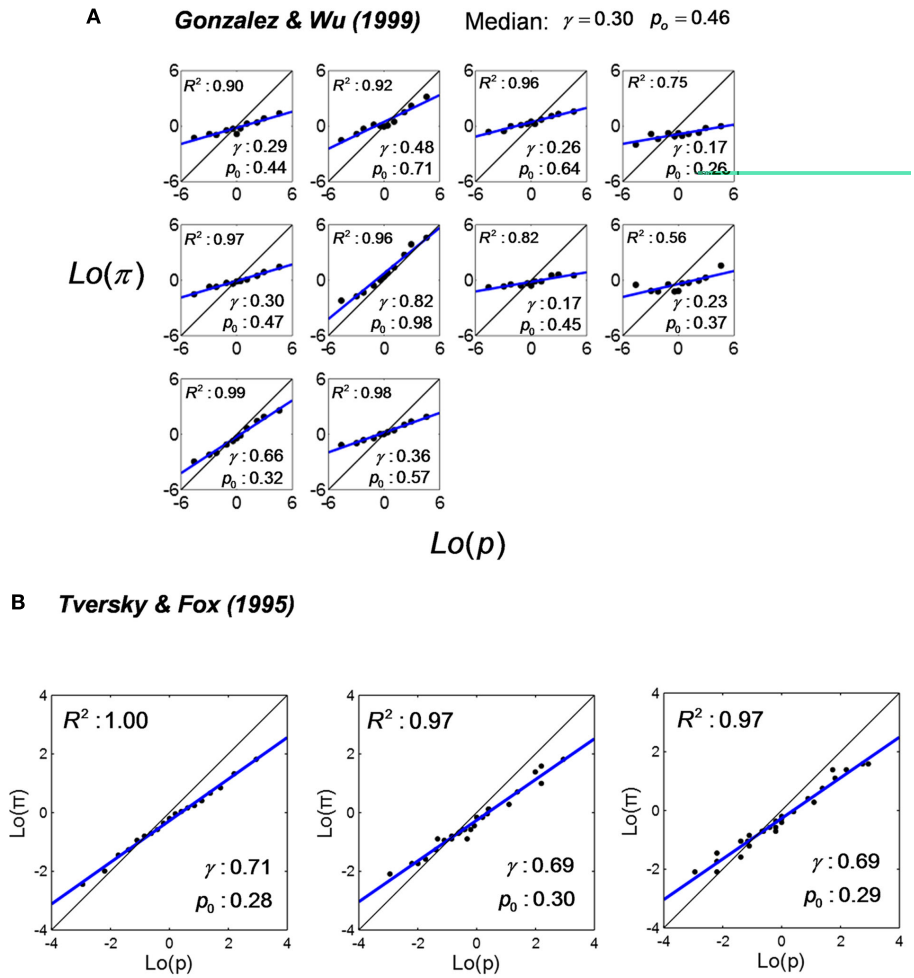

FIGURE 6 | Linear in log odds fits: decision under risk or uncertainty. Decision weight is plotted versus experimenter-stated probability (in decision under risk) or self-judged probability (in decision under uncertainty) and fitted by the LLO function. Black dots denote data. The blue line denotes the LLO fit. R^2 denotes the proportion of variance accounted by the fit. **(A)** Decision weights of individual participants from Gonzalez and Wu (1999). Each panel is for one participant. Participants chose between a two-outcome lottery and a sure reward. The probability of winning the larger reward of the lottery was stated as *p*. Decision weight, the counterpart of subjective probability π, was inferred from each participant's choices based on the Cumulative Prospect Theory. Re-plotted from Figure 6 of Gonzalez and Wu (1999). **(B)** Decision weights from Tversky and Fox (1995). Participants chose between a lottery offering a probability of a reward or otherwise zero and a sure reward. The probability of winning the larger reward of the lottery *p* was stated (left panel), or estimated by participants themselves as the probability of a specific Super Bowl prospect (middle panel), or as the probability of a specific Dow-Jones prospect (right panel). Decision weight, the counterpart of subjective probability π, was inferred from participants' choices based on the Cumulative Prospect Theory Re-plotted respectively from Figures 7–9 of Tversky and Fox (1995).

1964; ., 1990)
(, 1961; , 1966; B
M , 1977). D
(., E , 1964; , 1966). E LL ,
γ.
. A
I E 1,
. E
γ p0: *experience* ( ) *sample numerosity*, *N*,
200, 300, 400, 500, 600.

Methods

Participants. E , ,
. , fi , .
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-
$12/
C
J (CAIH)
.

Apparatus and Stimuli. -
.
GDM-F 900 24″ C D

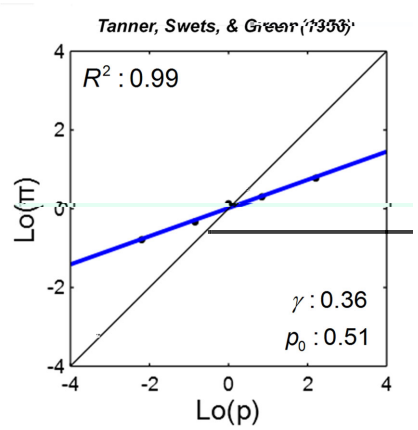

FIGURE 7 | Linear in log odds fit: signal detection theory. Estimated probability of signal present is plotted against the true probability on log odds scales for one participant. Black dots denote data. The blue line denotes the LLO fit. R^2 denotes the proportion of variance accounted by the fit. In Tanner et al. (1956), c.f. Green and Swets (1966/1974), participants were asked to report whether a sound signal was present or absent. Estimated probability was inferred from the participant's decision criterion based on signal detection theory. Data are from Table 4-1 of Green and Swets (1966/1974).

D 745 -
(B , 1997; , 1997). A

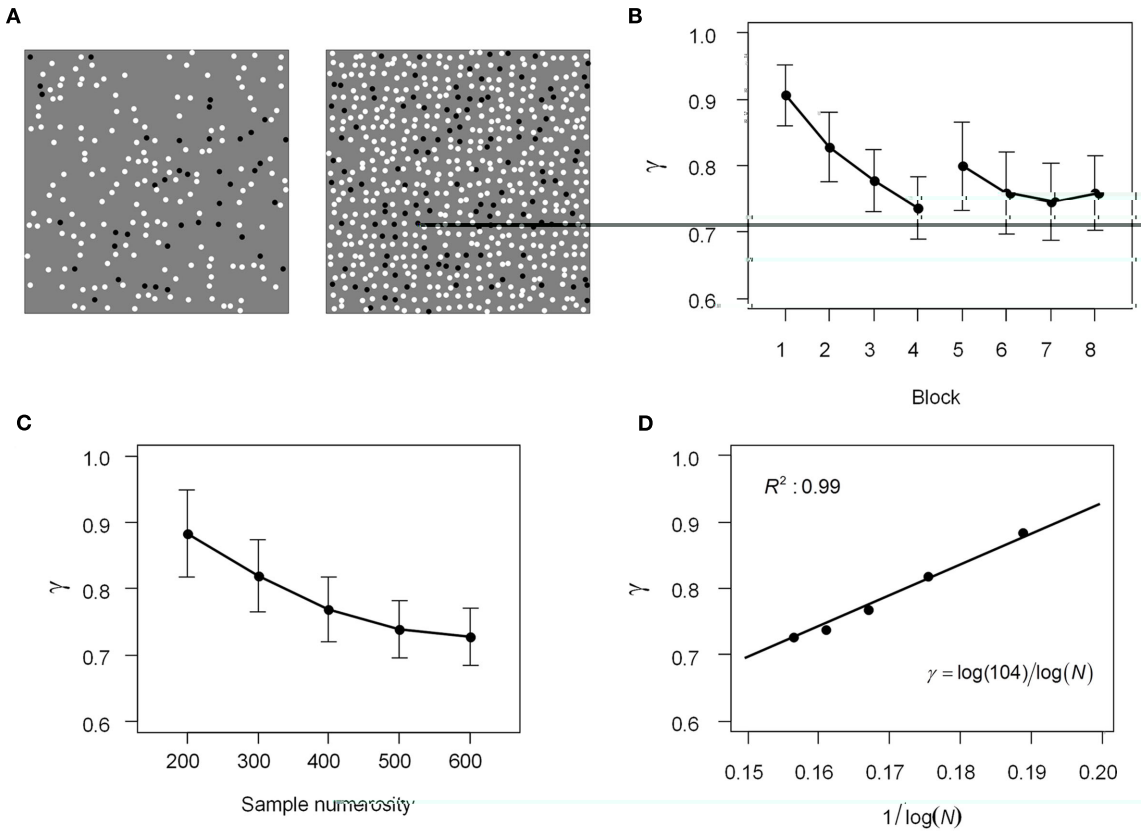

FIGURE 8 | Slope of distortion in relative frequency estimation. The methods and results of Experiment 1. **(A)** Examples of the relative frequency task: what proportion of the dots are black? The left display contains 200 dots in total, the right, 600. In both displays, 20% of the dots are black. **(B)** Effect of experience. The mean slope γ across 11 participants is plotted against block index, one to four for the first session, five to eight for the second session. Later blocks are supposed to be associated with more experience. More experience led to greater distortion (γ further from 1). Error bars denote SEs of the mean. **(C)** Effect of sample numerosity. The slope γ across 11 participants is plotted as a function of sample numerosity *N* (the total number of dots displayed in a trial). Larger sample numerosity resulted in greater distortion (γ further from 1). Error bars denote SEs of the mean. **(D)** The function of the mean γ to sample numerosity, *N*. Dots denote data. Solid line denotes the fit of γ as proportional to the reciprocal of log *N*.

$12/ .

CAIH

Apparatus and stimuli. E 1,

Procedure. I 3 . A fi . A E 1, 1 999 1000 . I - (0.1, 0.2, 0.3, 0.4) 0.02. . 400, 500, 600, 32 . E fi . , fi × 96 = 480 .

Results

F , 2003; ., 2006) 1/*m* 1/*m* .I , - - - , .

I 1/*m* , - . fi ( ) . 0.06, 0.36 . I fi 0.25, , fi . F , - ± 0.04 fi E . 1 .

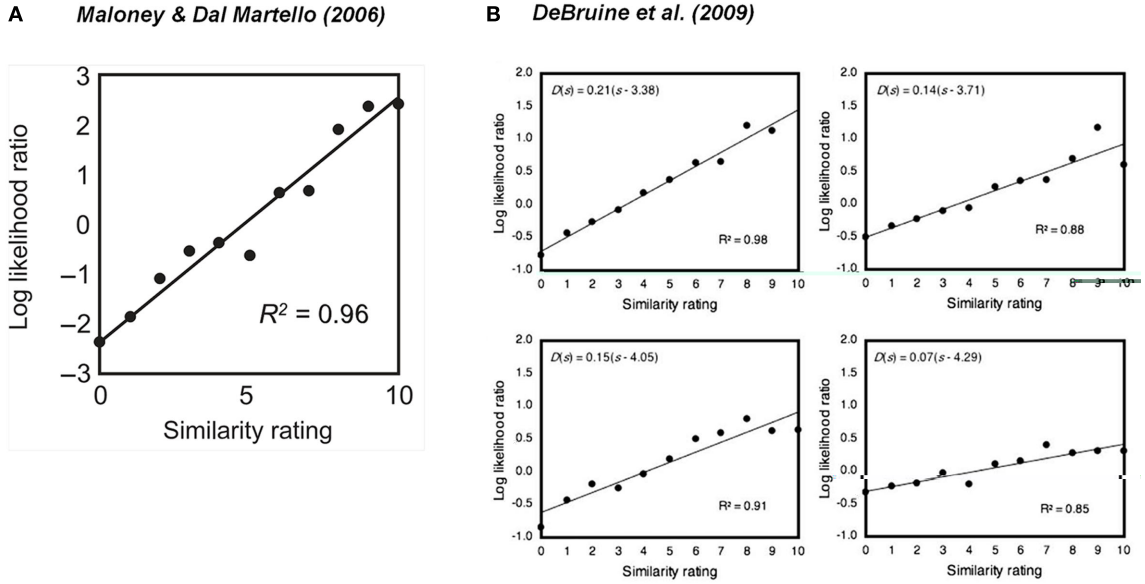

FIGURE 9 | Evidence for log odds as an inherent representation of uncertainty. Participants saw pairs of photos of faces. One group of participants rated the similarity between the two faces in each pair. A second group judged whether the two persons on each pair were related or not. **(A)** The similarity rating of two children faces is a linear transformation of the log odds of the two children being judged to be related. Reproduced from Maloney and Dal Martello (2006). **(B)** The similarity rating of two adult faces is a linear transformation of the log odds of the two adults being judged to be related. Reproduced from DeBruine et al. (2009). R^2 denotes the proportion of variance accounted by the linear fit. See text for implications.

F 10 , 95% - fi . 0.22 ± 0.07, 1/4 (0.25). 0.5. I 1/4 , . H - , fi , $r = 0.29$, $p = 0.42$. M , 0.18 ± 0.06, *lower* 1/4 (0.25).

$p_0 = 1/$, , .

EXPERIMENT 3: SLOPE AND DISCRIMINABILITY

K (1992) G (1999) - J . I E - 3, J . J - (J D) 0.5 fi E 1. I (fi), J D .

Methods

Participants. , , . E 1 2. J D. A J CAIH J .

Apparatus and stimuli. E 1.

Procedure. , 1.5 , 1 . H J , , , . A E 1, (*numerosity, N*) 200, 300, 400, 500, 600. . - J , J ±4%. fi 0.5. J - . F fi , 1- /2- 100 , 500 E 0.175, 0.1125, 0.0625, 0.05 , fi , , , . fi . F .

Results

1- /2- 70.7% J D . F , - fi - . 0.57, 0.57, 0.56, 0.56, 0.55, 200, 300, 400, 500, $12/ .

600. A - A A, - fi J D , $F(4,36) = 2.05$, $p = 0.11$, $\eta_p^2 = 0.18$. D - E 1.

DISCUSSION

A J L H J D , / J , γ p_0 (LL , E . 1). . I E 1 γ - . I , , . I , γ *N*. fi . H , - . I E 1 2 p_0 $p_0 = 1/$. F , 2003; ., 2006), $1/m$.

Decisions from experience

, - / - . (B E , 2003; H ., 2004; H F , 2009; J , 2009; , 2010), .

? A fi (. ., H ., 2004), (L , 2000). (H ., 2004; H F , 2009) J . C J , H . (2010) . , G . ( ) . I LL , ( ), (-).

E 1. A E . 4, fi γ, $N = C$, . J , fi .

. D - J E (1994) fi J , (100 J) (10 J). - J 60% 9/100 J 1/10 - J . p_0 . - (K , 1992; F , 1995; , 1998). J , $m > 2$, .

Confidence ratings

G (G ., 1991; G , 1994) - . fi , , . fi $\gamma > 1$ (**Figure 5**), - . J . , $\gamma > 1$. I - - fi .

PREVIOUS ACCOUNTS OF PROBABILITY DISTORTION

/ ? J , - . F , (L ., 1978) J (K , 1974). B . - (, 1991), J - , - - . - . - fi , fi . I , fl , .

FREQUENCY ESTIMATION

Power models

(1990) H D (2000)

fi

.1( )61.9 7(? D1800 7)-275

, , .
fi , , -
. .
(. ., $\gamma > 1$
) fi fi ,
- - - ,
- - -
fi . fi-
fi
fi fi . ,
fi .
. (1997) -
, , , $\gamma > 1$ -
.
H , fi
fi M G . (2004),
$\gamma > 1$ -
fi (**Figure 5B**).

DECISION UNDER RISK OR UNCERTAINTY

Adaptive probability theory

M (2006)
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fl B . I ,
fi fi .
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fl , M -
B . ,
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n, *a*,
b. B M (2006)
. M (2006) , -
, , . ., 0.1
$100, -
, .
B .

: . fi , *x*,

h_1 h_0 . fi , *y*.

y h_1

x h_0, . F

, G , ,

, G

h_1 h_0, fi

, $x - y$.

G

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. , fi

CONCLUDING REMARKS

L fi

. A 1884, J (1885)

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J

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I , K (1978, 1979) -

. G E (1987) fi K

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(F , 1995; G , 1999; K , 2001).

F ,

(G , 1966/1974; H K , 1981). I

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ACKNOWLEDGMENTS

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I H A . H

F .

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Conflict of Interest Statement: - fi - fl .

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